Math 10A
Worksheet, Final Review; Thursday, 8/9/2018
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1 Review Topics

1.1 Differentiation

- Domain/Range of functions
- Function transformations (Draw $f(2x+3)$)
- Limits
  - Infinite limits
  - L'Hopital's Rule
- Tangent Lines
  - Tangents to inverse functions
- Derivatives
  - Product Rule, Quotient Rule
  - Chain Rule
  - Implicit Differentiation
- Graphing Functions
  - Local extrema
  - Global extrema
  - Critical points
  - Concavity
  - Second Derivative Test
- Optimization
- Related Rates
- Taylor Series
- Newton's Method

1.2 Integration

- Antiderivatives
  - Fundamental Theorem of Calculus I and II
- Substitution Rule
- Integration by Parts
- Symmetry
- Numerical integration
  - Left/Right/Midpoint/Trapezoid/Simpson's Rule
  - Error Bounds
- Improper Integrals
  - Convergence Test
- Partial Fractions

1.3 Differential Equations

- Recurrence Relations
  - Going both forward and backward
  - Verifying solutions
- Identifying the adjectives (linear, homogeneous, etc.)
- Integrating Factors
- Separable Equations
- Second order differential equations
  - Going forward and backward
- IVPs/BVPs
- Slope fields
  - Euler's Method
  - Logistic Growth
- Linear systems of differential equations

1.4 Matrices

- Multiplying matrices, vectors
- Determinants
  - Number of solutions and how it depends on the determinant
- Gaussian Elimination
  - Finding Inverses
  - Solving matrix-vector equations
- Eigenvalues/eigenvectors
- Linear Regression
  - Least Squares Error
  - Finding line of best fit

2 Recurrence Relations and 2nd order Differential Equations

1. True False It is possible for an IVP to have a unique solution.
2. True False It is possible for a BVP to have a unique solution.
3. True False It is possible for an IVP to have infinitely many solutions.
4. True False It is possible for a BVP to have infinitely many solutions.
5. Solve the recursion equation $a_n = 2a_{n-2} - a_{n-1}$ with the initial conditions $a_0 = 0, a_1 = 3$.
6. Verify that $y_1(t) = t$ and $y_2(t) = t^3$ are solutions to the differential equation $t^2y''(t) - 3ty'(t) + 3y(t) = 0$. Find the solution to the differential equation with $y(1) = 2$ and $y'(1) = 4$ (hint: what kind of differential equation is this?).
7. Find all solutions to the BVP $y'' + 2y' + 5y = 0$ with $y(0) = 0$ and $y(\pi) = 0$.
8. Find all solutions to the BVP $y'' - 5y' + 6y = 0$ with $y(0) = 2$ and $y(1) = e^2 + e^3$.
9. Solve the initial value problem $3y'' + 18y' + 27y = 0$ with $y(0) = 0, y'(0) = 1$.
10. Solve the initial value problem given by $2y'' = 3y' - y$ and $y(0) = 0$ and $y'(0) = 1$.
11. Find a second order differential equation IVP that has te^t as a solution.
12. Find a second order differential equation BVP that has $e^{2t}\sin(t)$ as a solution.

13. Find the second order linear ODE such that $y(t) = te^{2t}$ is a solution to it.

14. What is the largest value of $\alpha > 0$ such that any solution of $y'' + 4y' + \alpha y = 0$ does not oscillate (does not have any terms of $\sin, \cos$).

3 First Order Differential Equations

15. Solve the differential equations $(t^3 + t^2)y' = \frac{t^2+2t+2}{2y}$ with the initial condition $y(1) = 1$.

16. Consider the differential equation $ty' + 3y = 5t^2$ with initial condition $y(1) = 1$. Draw a slope field and then estimate $y(5)$ using a step size of $h = 2$. Then solve for y explicitly and find the exact value of $y(5)$.

17. For more Euler's method practice, see the Discussion 29 Worksheet.

18. Find all solutions to $e^t y' = y^2 + 2y + 1$.

19. Find the solution of $y' + \frac{y}{x} = e^x/x$ with $y(1) = 0$.

20. Find the solution to $r' = r^2/t$ with $r(1) = 1$.

21. Find the general solution to $y' + 2y/x = \sin(x)/x^2$.

22. Find the general solution of $y' = 2t \sec y$.

23. Find the general solution to $y' - 2y/x = 3x^3$.

4 Matrices

24. True False If A, B are square $n \times n$ matrices, then $AB = BA$.

25. True False If A is a 2×2 matrix such that $A^2 = I_2$, then $A = I_2$.

26. Find the general solution to the systems of linear differential equations

$$\begin{cases} y_1'(t) = 2y_1(t) + y_2(t) \\ y_2'(t) = y_1(t) + 2y_2(t) \end{cases}$$

27. Find the general solution to the systems of linear differential equations

$$\begin{cases} y_1'(t) = y_1(t) + 4y_2(t) \\ y_2'(t) = 2y_1(t) + 3y_2(t) \end{cases}$$

28. Find the general solution to the systems of linear differential equations

$$\begin{cases} y_1'(t) = -y_1(t) + 3y_2(t) \\ y_2'(t) = 2y_1(t) \end{cases}$$

29. Consider the following set of points: $\{(0,6),(1,3),(2,1),(3,0),(4,0)\}$. Find the line of best fit through these points and use it to estimate $y(0.5)$.

30. Let $A = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 2 \end{pmatrix}$. Find A^{-1}.

31. Let $A = \begin{pmatrix} 3 & 4 & -1 \\ 4 & 2 & 1 \\ -2 & -3 & 1 \end{pmatrix}$. Find A^{-1}.

32. Let A be the same as the previous problem. Solve $A\vec{x} = \begin{pmatrix} 1 \\ 4 \\ -1 \end{pmatrix}$ (hint: use the previous problem to do this quickly).

33. Let $\vec{v}_1 = \begin{pmatrix} 3 \\ -2 \\ 2 \end{pmatrix}, \vec{v}_2 = \begin{pmatrix} -1 \\ -1 \\ 0 \end{pmatrix}, \vec{v}_3 = \begin{pmatrix} -1 \\ 1 \\ -2 \end{pmatrix}$ and suppose that A is a 3×3 matrix such that $A\vec{v}_1 = 4\vec{v}_1, A\vec{v}_2 = \vec{0}, A\vec{v}_3 = -\vec{v}_3$. What are the eigenvalues and eigenvectors of A? What is the general solution to $\vec{y}'(t) = A\vec{y}$ with $\vec{y} = \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix}$?

34. Let A be a 2×2 matrix and suppose that $\vec{y} = \begin{pmatrix} 3e^{2t} + 4e^{4t} \\ e^{4t} - e^{2t} \end{pmatrix}$ is a solution to $\vec{y}' = A\vec{y}$. What are the eigenvalues and eigenvectors of A? What is $A\begin{pmatrix} 3 \\ -1 \end{pmatrix}, A\begin{pmatrix} 4 \\ 1 \end{pmatrix}$ and $A\begin{pmatrix} 7 \\ 0 \end{pmatrix}$?

35. Find the line of best fit through the points $\{(0,2),(1,3),(2,1)\}$.

36. Write the differential equation $y'' + 5y' + 6y = 0$ as a systems of differential equations with $y_1(t) = y(t), y_2(t) = y'(t)$ and solve with $y(0) = 2, y'(0) = -5$.